BIMETRIC THEORY OF FRACTIONAL QUANTUM HALL STATES

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AG, Dam Thanh Son 1705:06739 (To appear in PRX)

Dung Nguyen, **AG,** Dam Thanh Son (In Progress)

Other major references

Chui Phys. Rev. B 34, 1409 - **(1986)**

Tokatly Phys. Rev. B 73, 205340, Phys. Rev. B 74, 035333 - **(2006)**

Haldane arXiv:0906.1854, PRL (107) 116801, arXiv:1112.0990 - **(2009-2011)**

Maciejko, Hsu, Kivelson, Park, Sondhi PRB 88, 125137 - **(2013)**

You, Cho, Fradkin PRX 4 041050 - **(2014)**

PLAN

Introduction to QH effect in curved space

Girvin-MacDonald-Platzman mode

- Lowest Landau Level
- W_∞ algebra
- Single Mode Approximation

Bimetric theory of FQH states

- Bimetric theory
- How does it work ?
- Consistency checks

Conclusions and open directions

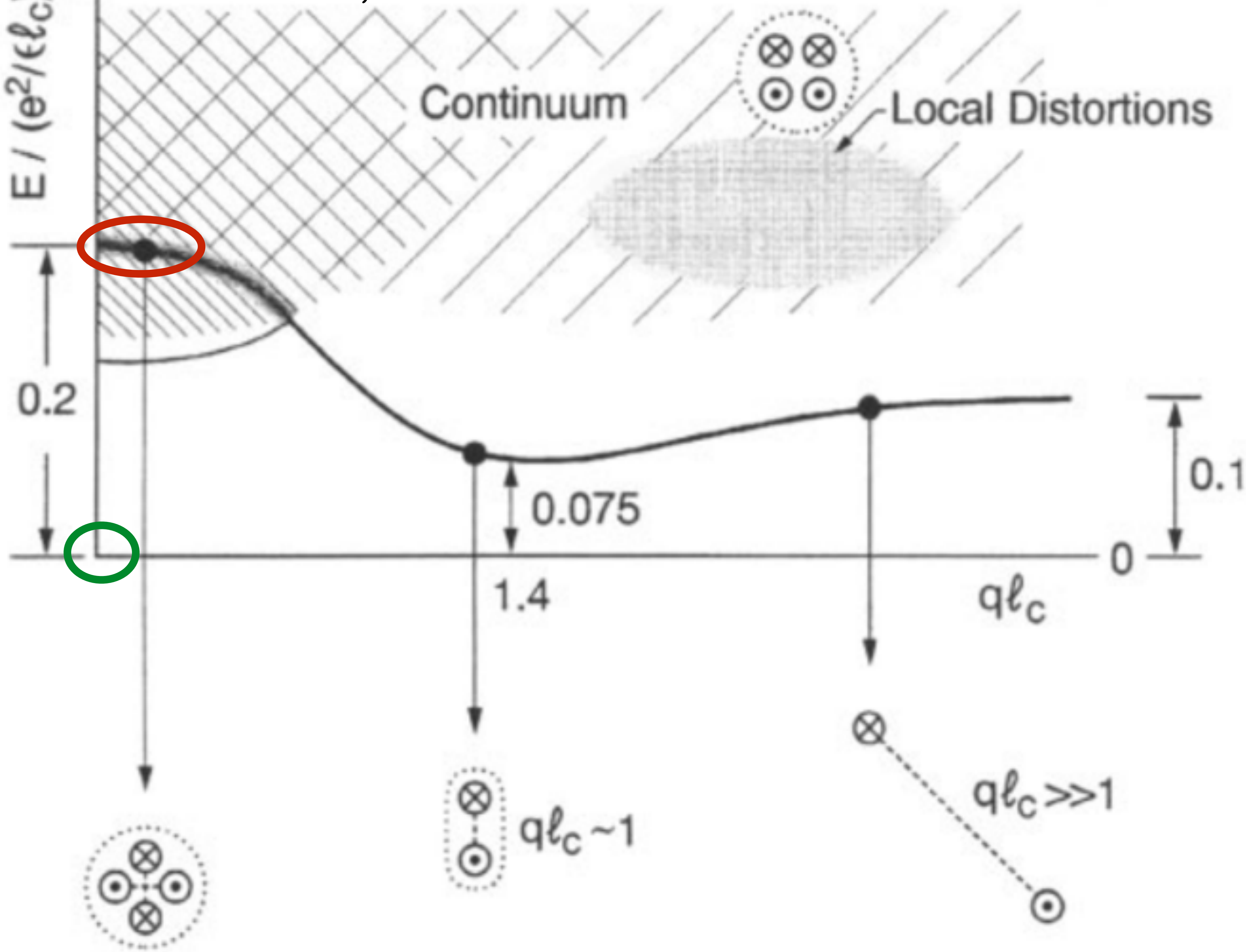
AT THE QUANTUM HALL PLATEAU

- Gap to all excitations (charged *and* neutral)
- All dissipative transport coefficients vanish
- Parity and time-reversal broken, but $\mathcal{PT}$-symmetric
- *No* Lorentz invariance
- Quantized non-dissipative transport coefficients
- *Not* uniquely characterized by the filling factor

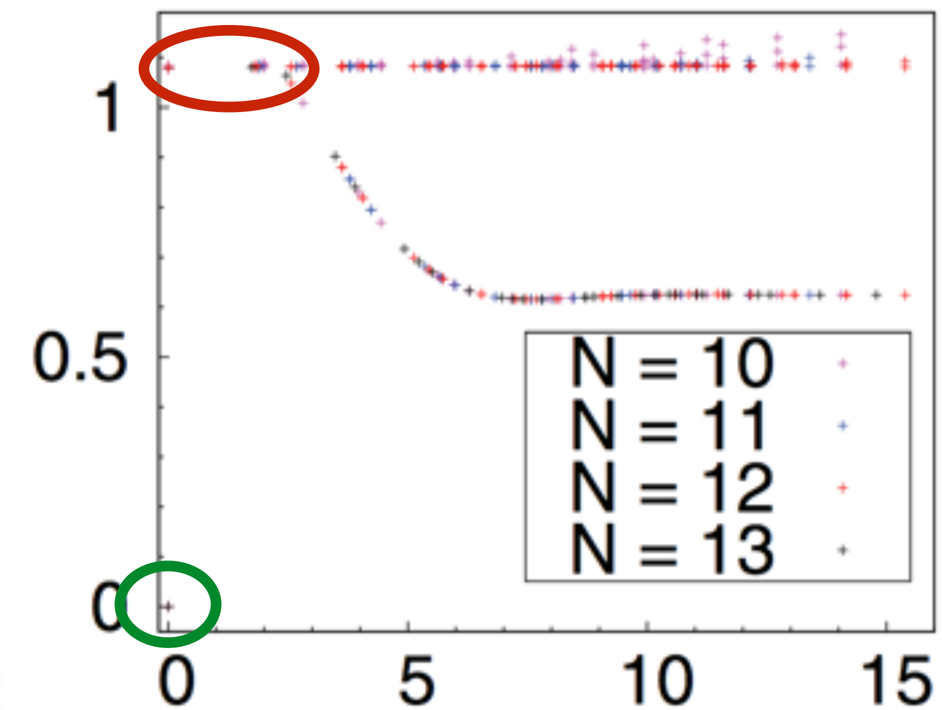
$$N = \nu N_\phi \qquad \sigma_{xy} = \nu \frac{e^2}{h}$$

SPECTRUM

Platzman, He 1996

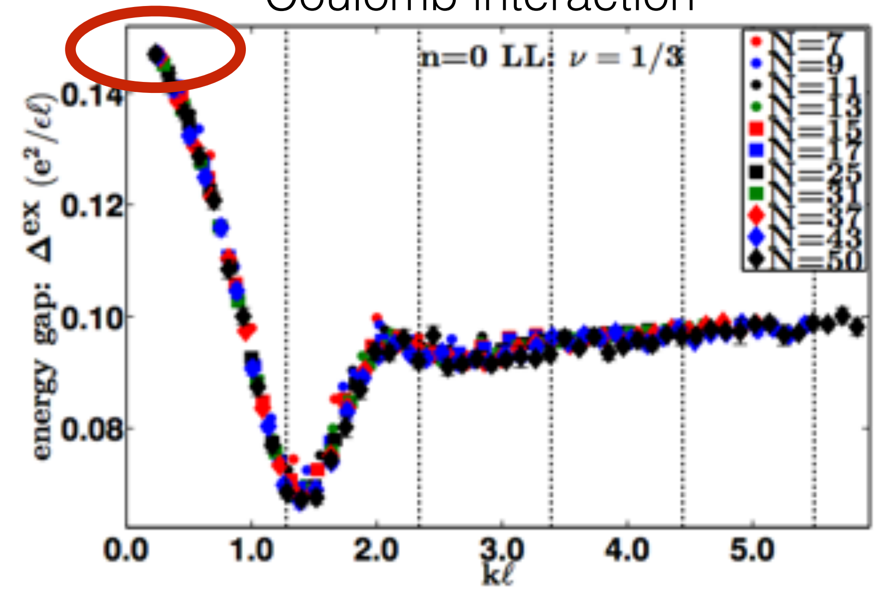


Short-ranged interaction



Repellin, Neupert, Papic, Regnault 2014

Coulomb interaction



Balram, Pu 2017

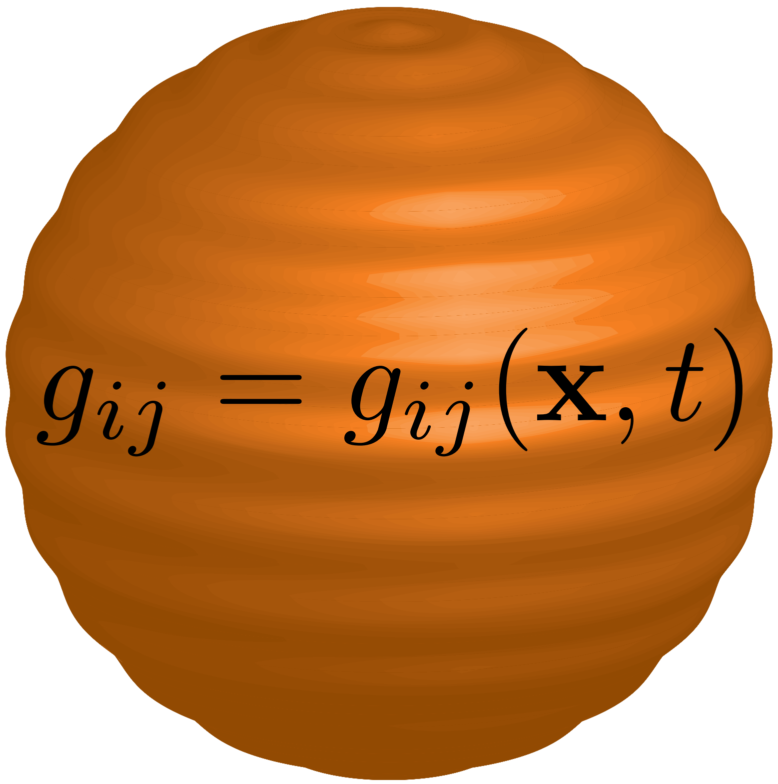
Does anything universal happen at the scale $E \sim gap$?

Girvin, MacDonald, Platzman 1986

Haldane Rezayi 1985

GEOMETRY

Geometry is encoded into time-dependent metric



$$ds^2 = g_{ij}(\mathbf{x}, t)dx^i dx^j$$

It is more convenient to use vielbeins

$$g_{ij} = e_i^A e_j^B \delta_{AB} \qquad \mathbf{g} = \mathbf{e} \cdot \mathbf{e}^T$$

There is a $SO(2)$ redundancy

Corresponding ``gauge field'' is the *spin* connection ω_μ

Spin connection is a ``vector potential'' for curvature

$$\frac{R}{2} = \partial_1 \omega_2 - \partial_2 \omega_1 \qquad \omega_0 \sim \epsilon_A{}^B e_B^i \partial_0 e_i^A$$

CHERN - SIMONS THEORY OF FQH STATES

Determines filling $\nu = k^{-1}$

electric charge of constituent particles

``mean orbital spin''

Wen-Zee term

$$S = \frac{k}{4\pi}\int ada - \frac{q}{2\pi}\int adA - \frac{s}{2\pi}\int ad\omega$$

quantum ``emergent'' gauge field

external e/m field

$SO(2)$ spin connection

For multi-component states each component has its own s_I

Wen, Zee 1991

WEN - ZEE TERM

Wen-Zee term couples the electron density to curvature

$$\rho = \frac{\nu}{2\pi} B + \frac{\nu s}{4\pi} R$$

Implies a global relation on a compact Riemann surface

$$N = \nu N_\phi + \nu \mathcal{S} \frac{\chi}{2}$$

← Euler characteristic

Quantum number $\mathcal{S} = 2s$ is called *Shift*

Also describes the quantum Hall viscosity

$$\langle T_{xx} T_{xy} \rangle = i\omega \eta_H \qquad \eta_H = \hbar \frac{\mathcal{S}}{4} \bar{\rho}$$

Haldane 1983 Wen, Zee 1991 Avron Zograf Seiler 1995 Read 2009

BEYOND TQFT TOOLS

Beyond TQFT we face a strongly interacting problem

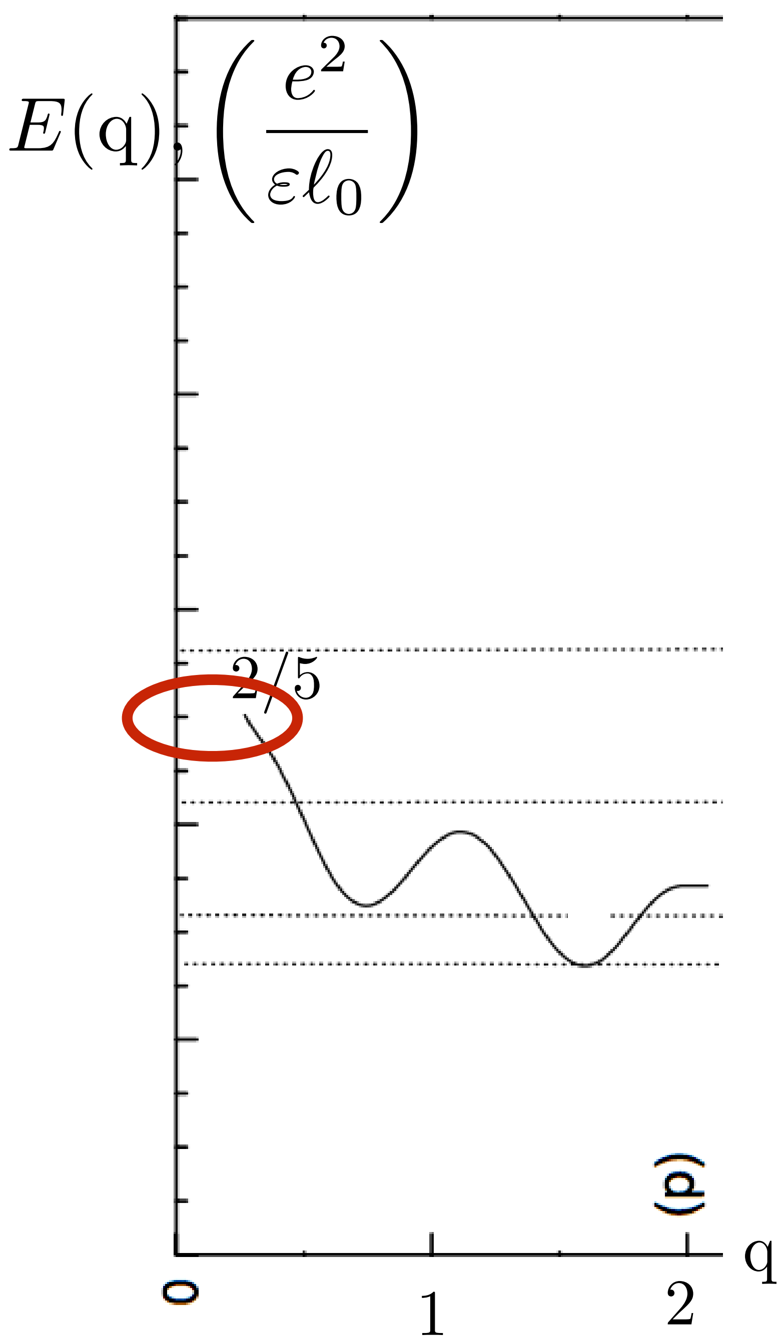
What can we do about it ?

- Trial states
- Exact diagonalization
- Hydrodynamics
- Flux attachment (composite bosons and fermions)
- *Bimetric theory*

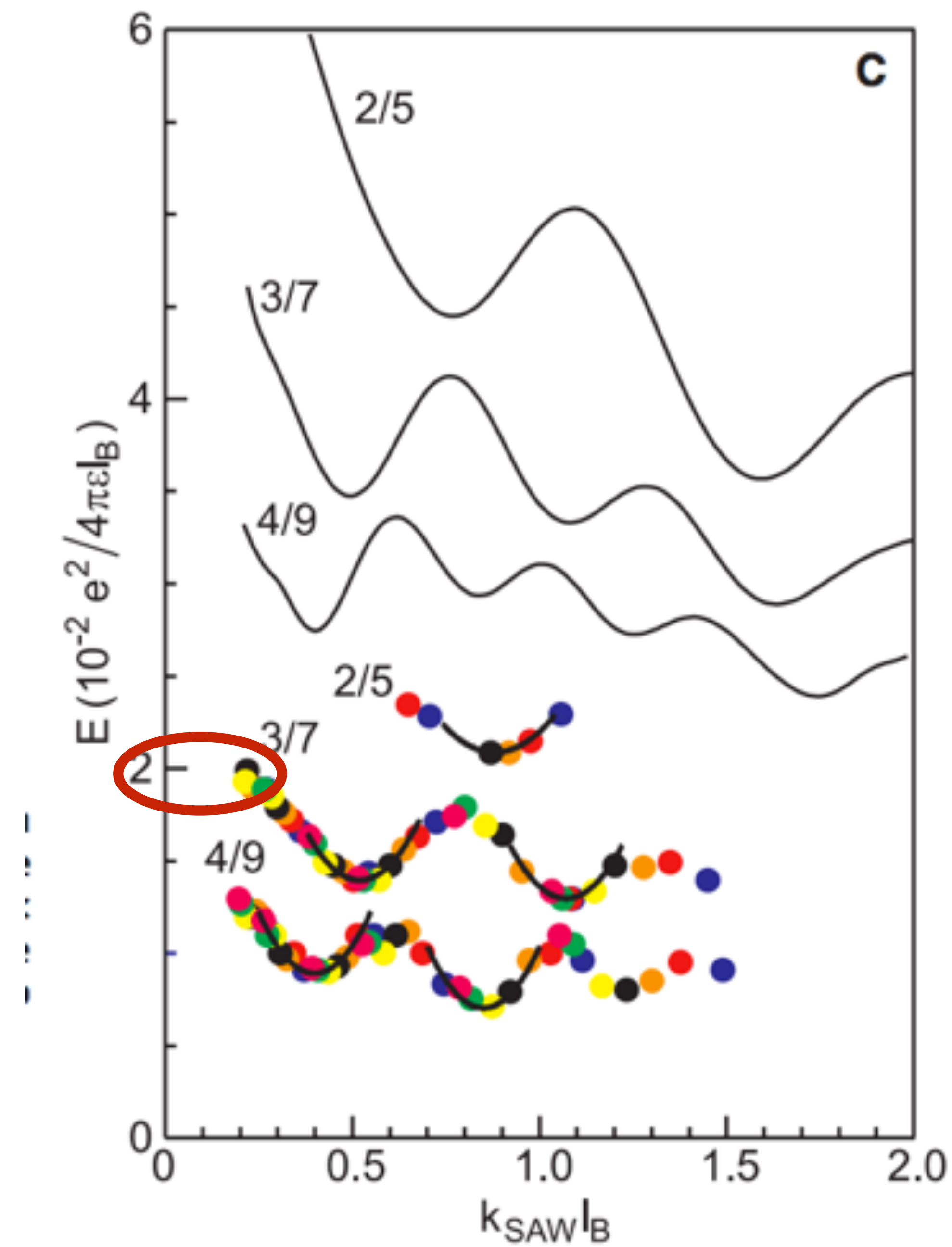
GIRVIN - MACDONALD - PLATZMAN STATE

GMP MODE IN EXPERIMENT

The GMP mode has been observed in inelastic light scattering experiments



Kang, Pinczuk, Dennis, Pfeiffer, West 2001



Kukushkin, Smet, Scarola, Umansky, von Klitzing 2009

GENERAL REMARKS ABOUT THE GMP MODE

★ Universally present in **fractional** QH states

★ Absent in **integer** QH states

★ Angular momentum or ``spin'' **2**, regardless of microscopic details

★ Nematic phase transition = condensation of the GMP mode

★ Effective theory of the GMP mode should to be a *theory of massive spin-2 excitation*

GIRVIN - MACDONALD - PLATZMAN ALGEBRA

The electron density operator

$$\rho(\mathbf{x}) = \sum_{i=1}^{N_{\rm el}} \delta(\mathbf{x} - \mathbf{x}_i) \quad \xrightarrow{\text{Fourier}} \quad \rho(\mathbf{k}) = \frac{1}{2\pi} \sum_{i=1}^{N_{\rm el}} e^{i\mathbf{k}\cdot\mathbf{x}_i}$$

In complex coordinates $\mathbf{k} \cdot \mathbf{x}_i = \bar{k} z_i + k \bar{z}_i$

After the Lowest Landau Level projection $\bar{z} \longrightarrow 2\partial_z$

Projected density operators $:\bar{\rho}(\mathbf{k}):= \sum_{i=1}^{N_{\rm el}} e^{ik\partial_{z_i}} e^{i\bar{k}z_i}$

Satisfy W_∞ algebra

$$[\bar{\rho}(\mathbf{k}), \bar{\rho}(\mathbf{q})] = 2i \sin\left[\frac{\ell^2}{2}\mathbf{k} \times \mathbf{q}\right] \bar{\rho}(\mathbf{k} + \mathbf{q})$$

GIRVIN - MACDONALD - PLATZMAN MODE I

Warning: not standard presentation

The LLL generators of W_∞ are $\mathcal{L}_{n,m} = \sum_{i=1}^{N_{\rm el}} z_i^{n+1} \partial_{z_i}^{m+1}$

LLL Rotation

LLL Shears, spin-2 operators

Operators $\{\mathcal{L}_{0,0}, \mathcal{L}_{1,-1}, \mathcal{L}_{-1,1}\}$ form $\mathfrak{sl}(2,\mathbb{R})$ algebra

The projected density operator is expanded in $\mathcal{L}_{n,m}$

$$\bar{\rho}(\mathbf{k}) = e^{-\frac{|k|^2}{2}} \sum_{m,n} c_{nm} \bar{k}^n k^m \mathcal{L}_{n-1,m-1}$$

$\mathcal{L}_{n,m}$ create *intra*-LL state at momentum $\mathbf{k}$

GIRVIN - MACDONALD - PLATZMAN MODE II

At long wave-lengths the GMP mode is

$$\bar{\rho}(\mathbf{k})|0\rangle = \left[\frac{\mathrm{k}^2}{8}\mathcal{L}_{-1,1} + \frac{\bar{\mathrm{k}}^2}{8}\mathcal{L}_{1,-1} + \ldots\right]|0\rangle$$

The GMP state $\bar{\rho}(\mathbf{k})|0\rangle$ is a shear distortion at small $\mathbf{k}$

For IQH $\bar{H} = 0$ ⟶ $\bar{\rho}(\mathbf{k})|0\rangle$ is a 0 energy state

Consider two-body Hamiltonian $\bar{H} = \sum_{\mathbf{k}} V(\mathbf{k})\bar{\rho}(-\mathbf{k})\bar{\rho}(\mathbf{k})$

Since $[H, \bar{\rho}(\mathbf{k})] \neq 0$ the shear distortion costs energy

At small $\mathbf{k}$ GMP mode is a gapped, propagating, shear distortion of the FQH fluid

BIMETRIC THEORY

BIMETRIC GEOMETRY

The spin-2 mode is described by a symmetric matrix $\mathfrak{h}_{AB}(\mathbf{x}, t)$

Given $\mathfrak{h}_{AB}$ we introduce an ``intrinsic'' metric and vielbein

$$\hat{g}_{ij} = e_i^A e_j^B \mathfrak{h}_{AB} = \hat{e}_i^\alpha \hat{e}_j^\beta \delta_{\alpha\beta}$$

FQH constraint: $\sqrt{g} = \sqrt{\hat{g}}$

$\widehat{SO}(2)$ spin connection and curvature follow

$$\frac{\hat{R}}{2} = \partial_1 \hat{\omega}_2 - \partial_2 \hat{\omega}_1 \qquad \hat{\omega}_0 = \frac{1}{2} \epsilon^\alpha{}_\beta \hat{e}^i_\alpha \partial_0 \hat{e}^\beta_i$$

Not the same as two copies of Riemannian geometry

$$\mathrm{Diff} \times \widehat{\mathrm{Diff}} \to \mathrm{Diff}_{\mathrm{diag}}$$

This geometry involves two metrics $(g_{ij}, \hat{g}_{ij})$, hence *bimetric**

*Appeared recently in theories of massive gravity

de Rham, Gabadadze, Tolley
2010

VISUALISATION

Can visualize $\hat{g}_{ij}$ as a (fluctuating) pattern on the surface

BIMETRIC THEORY

Chern-Simons theory interacting with fluctuating metric

Pronounced: ``sigma''

$$\mathcal{L} = \frac{k}{4\pi} ada - \frac{1}{2\pi} Ada - \frac{s}{2\pi} ad\omega - \frac{\varsigma}{2\pi} ad\hat{\omega} + S_{\text{pot}}[\hat{g}]$$

We integrate out the gauge field

$$\mathcal{L} = \mathcal{L}_1[A, g] + \mathcal{L}_{bm}[\hat{g}; A, g]$$

Where $\mathcal{L}_1[A, g]$ contains no dynamics and

$$\mathcal{L}_{\text{bm}} = \frac{\nu\varsigma}{2\pi} A d\hat{\omega} - \frac{M}{2} \left(\frac{1}{2} \hat{g}_{ij} g^{ij} - \gamma \right)^2$$

For IQH $k = 1$ there is no *intra-LL dynamics and*

$$\mathcal{L} = \mathcal{L}_1[A; g]$$

AG, Son 2017

GEOMETRIC OPERATORS

Density and current operators acquire geometric meaning

Fluctuations of electron density = fluctuations of local Ricci curvature

$$\rho = \frac{\nu \varsigma}{4\pi} \hat{R}$$

Fluctuations of electron current = fluctuations of ``gravi-electric'' field

$$j^i = \frac{\nu \varsigma}{2\pi} \epsilon^{ik} \hat{\mathcal{E}}_k$$

To the leading order in $\mathbf{k}$, *everything* is determined by ς

Continuity equation holds identically

$$\partial_0 \hat{R} + \epsilon^{ik} \partial_i \hat{\mathcal{E}}_k \equiv 0$$

POTENTIAL TERM

$$\mathcal{L}_{\text{pot}} = -\frac{M}{2}\left(\frac{1}{2}\hat{g}_{ij}g^{ij} - \gamma\right)^2 = -\frac{M}{2}\left(\frac{1}{2}\text{Tr}(\mathfrak{h}) - \gamma\right)^2$$

This potential has two phases

If $\gamma < 1$ the theory is in the gapped ``symmetric'' phase

$$\mathfrak{h}_{AB} = \delta_{AB}\,, \qquad \hat{g}_{ij} = g_{ij}$$

If $\gamma > 1$ the theory is in the gapless nematic phase

$$\mathfrak{h}_{AB} = \mathfrak{h}^{(0)}_{AB} \qquad \hat{g}_{ij} \neq g_{ij}$$

We will be interested in the ``symmetric'' phase

LINEARIZATION

In flat space we chose the parametrization

$$\mathfrak{h}_{AB} = \exp\begin{pmatrix} Q_2 & Q_1 \\ Q_1 & -Q_2 \end{pmatrix}, \qquad \begin{aligned} Q &= Q_1 + iQ_2 \\ \bar{Q} &= Q^* \end{aligned}$$

and linearize in *flat* space around

$$Q = 0\,, \quad \mathfrak{h}_{AB} = \delta_{AB}$$

to find

Gap of the GMP mode

$$\mathcal{L}_{\mathrm{bm}} \approx i\frac{\varsigma\rho_0}{4}\bar{Q}\dot{Q} - \frac{m}{2}|Q|^2$$

Maciejko, Hsu, Kivelson, Park, Sondhi 2013

You, Cho, Fradkin 2014

STATIC STRUCTURE FACTOR

To determine the coefficient ς we calculate the SSF

$$\bar{s}(\mathbf{k}) = 2\pi\ell^2\nu^{-1}\langle\bar{\rho}_{-\mathbf{k}}\bar{\rho}_{\mathbf{k}}\rangle$$

Calculation in the linearized theory reveals

$$\bar{s}(\mathbf{k}) = \frac{2|\varsigma|}{8}|\mathbf{k}|^4 + \dots$$

Match this to a general LLL result for chiral states

$$\bar{s}(\mathbf{k}) = \frac{|\mathcal{S} - 1|}{8}|\mathbf{k}|^4 + \dots$$

This uniquely determines

$$2|\varsigma| = |\mathcal{S} - 1|$$

Vanishes for IQH

CANONICAL QUANTIZATION

Turn off external fields

Invariant under $SL(2,\mathbb{R})$

Potential breaks $SL(2,\mathbb{R})$

$$\frac{\nu\varsigma}{2\pi}Ad\hat{\omega} = \frac{\nu\varsigma}{2\pi}B\hat{\omega}_0 = \frac{\varsigma\rho_0}{2}\epsilon_\alpha{}^\beta \hat{e}^i_\beta \frac{\partial}{\partial t}\hat{e}^\alpha_i$$

From the action we read the CCR

$$[\hat{e}^i_\alpha(\mathbf{x}), \hat{e}^\beta_j(\mathbf{x}')] = -\frac{2i}{\rho_0\varsigma}\delta^i_j\epsilon_\alpha{}^\beta\delta(\mathbf{x}-\mathbf{x}')$$

Which leads to the $\mathfrak{sl}(2,\mathbb{R})$ algebra for the metric

$$[\hat{g}_{zz}(\mathbf{x}), \hat{g}_{\bar{z}\bar{z}}(\mathbf{x}')] = \frac{16}{\rho_0\varsigma}\hat{g}_{z\bar{z}}(\mathbf{x})\,\delta(\mathbf{x}-\mathbf{x}')$$

$$[\hat{g}_{\bar{z}\bar{z}}(\mathbf{x}), \hat{g}_{z\bar{z}}(\mathbf{x}')] = \frac{8}{\rho_0\varsigma}\hat{g}_{\bar{z}\bar{z}}(\mathbf{x})\delta(\mathbf{x}-\mathbf{x}')$$

Haldane 2011

Appeared in Verlinde 1989

GMP ALGEBRA

Algebra of the spin connections closes

$$[\hat{\omega}_i(\mathbf{k}),\, \hat{\omega}_j(\mathbf{q})] = \frac{1}{\rho_0 \varsigma}\Big[k_j \hat{\omega}_i(\mathbf{k}+\mathbf{q}) - q_i \hat{\omega}_j(\mathbf{k}+\mathbf{q})\Big] - \frac{i\epsilon_{ij}}{2\rho_0\varsigma}\hat{R}(\mathbf{k}+\mathbf{q})$$

Spin connection couples like the dipole moment

$\hat{\omega} dA \approx E_i \epsilon_{ij} \hat{\omega}_j = \mathbf{E}\cdot(\epsilon\hat{\omega})$ (comare to $\mathrm{U} = -\mathbf{E}\cdot\mathbf{d}$)

``Dipole'' algebra implies

$$[\hat{R}(\mathbf{k}), \hat{R}(\mathbf{q})] = \frac{4\pi}{\nu\varsigma} i(\mathbf{k}\times\mathbf{q})\ell^2 \hat{R}(\mathbf{k}+\mathbf{q})$$

Small $\mathbf{k}$ GMP algebra follows $[\bar{\rho}(\mathbf{k}), \bar{\rho}(\mathbf{q})] \approx i\ell^2(\mathbf{k}\times\mathbf{q})\,\bar{\rho}(\mathbf{k}+\mathbf{q})$

AG, Son 2017

WHAT ELSE CAN BIMETRIC THEORY DO ?

Complete Lagrangian up to three derivatives

$$\mathcal{L}_{\rm bm} = \frac{\nu\varsigma}{2\pi} A d\hat{\omega} - \frac{\hat{c}}{4\pi}\hat{\omega} d\hat{\omega} - \frac{\nu\varsigma}{4\pi}\hat{\nabla}_i E_i B - \frac{\hat{c}\ell^2}{8\pi}\hat{\nabla}_i E_i \hat{R} - \frac{\tilde{m}}{2}\left(\frac{1}{2}\hat{g}_{ij}g^{ij} - \gamma\right)^2 - \frac{\alpha}{4}\left|\Gamma - \hat{\Gamma}\right|^2$$

- ★ *Projected* static structure factor up to $|\mathbf{k}|^6$
- ★ Dispersion relation of the GMP mode up to $|\mathbf{k}|^2$
- ★ *Absence* of the GMP mode and nematic transition in IQH
- ★ Hidden LLL projection and manifest Particle-Hole duality
- ★ Girvin-MacDonald-Platzman algebra holds up to $|\mathbf{k}|^4$
- ★ ``Guiding center'' DC Hall conductivity to $|\mathbf{k}|^2$
- ★ ``Guiding center'' Hall viscosity to $|\mathbf{k}|^2$
- ★ Shear modulus of the FQH fluid
- ★ Hints at rich structure of the full W_∞ theory and more…

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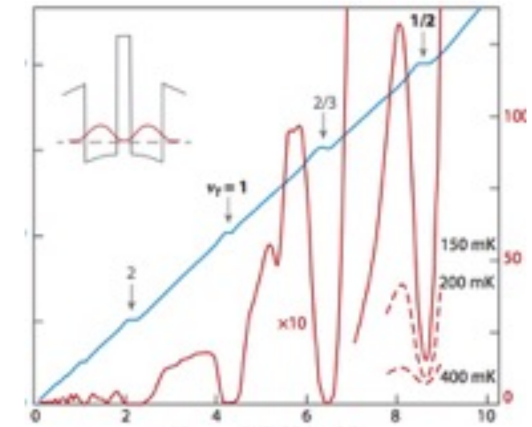
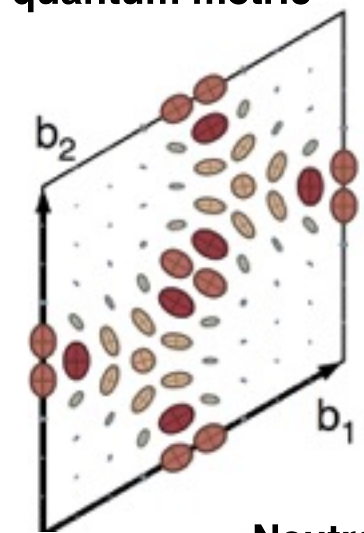
OPEN PROBLEMS

- ★ Understand the non-linearity
- ★ Fully covariant formulation
- ★ Implications for the boundary theory
- ★ CFT construction of the GMP state?
- ★ Competing orders in multi-layer states
- ★ Non-linear higher spin theory
- ★ Fractional Chern insulators
- ★ Collective neutral fermion mode in 5/2 state
- ★ Bimetric theory for PH-Pfaffian
- ★ Covariant, nonlinear formulation of CFL
- ★ Quantum Hall liquid crystal phases
- ★ Detailed study of anisotropic FQH states
- ★ Relation to ``fracton'' theories?
- ★ 3D
- ★ …….

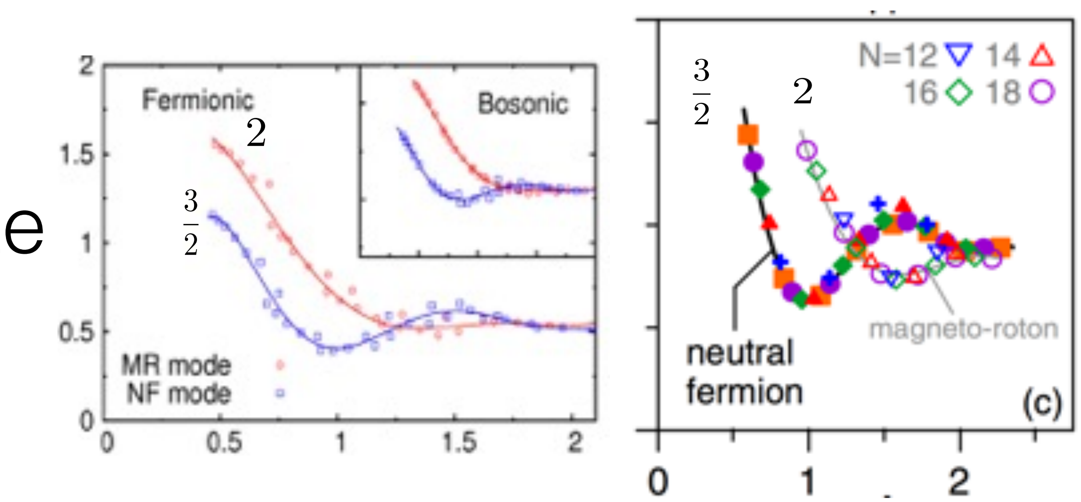
Bi-layer FQH



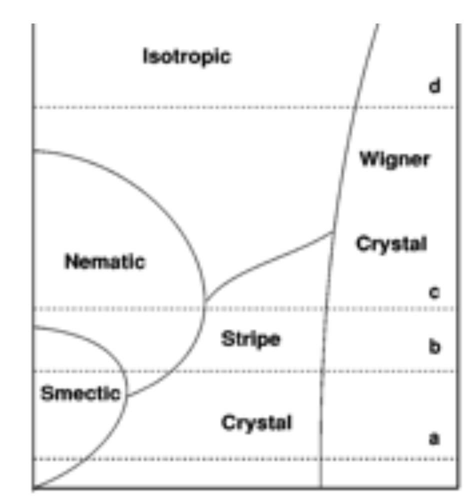
FCI
``quantum metric''



Neutral Fermion in 5/2 state



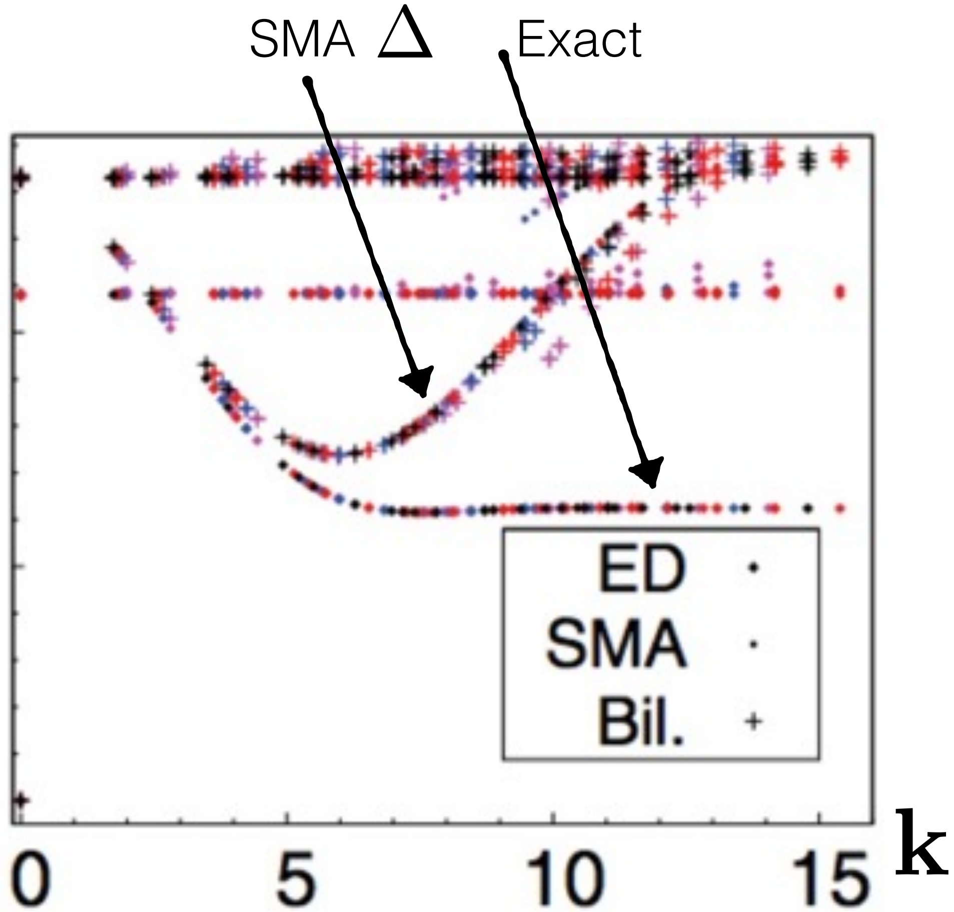
FQH liquid crystal



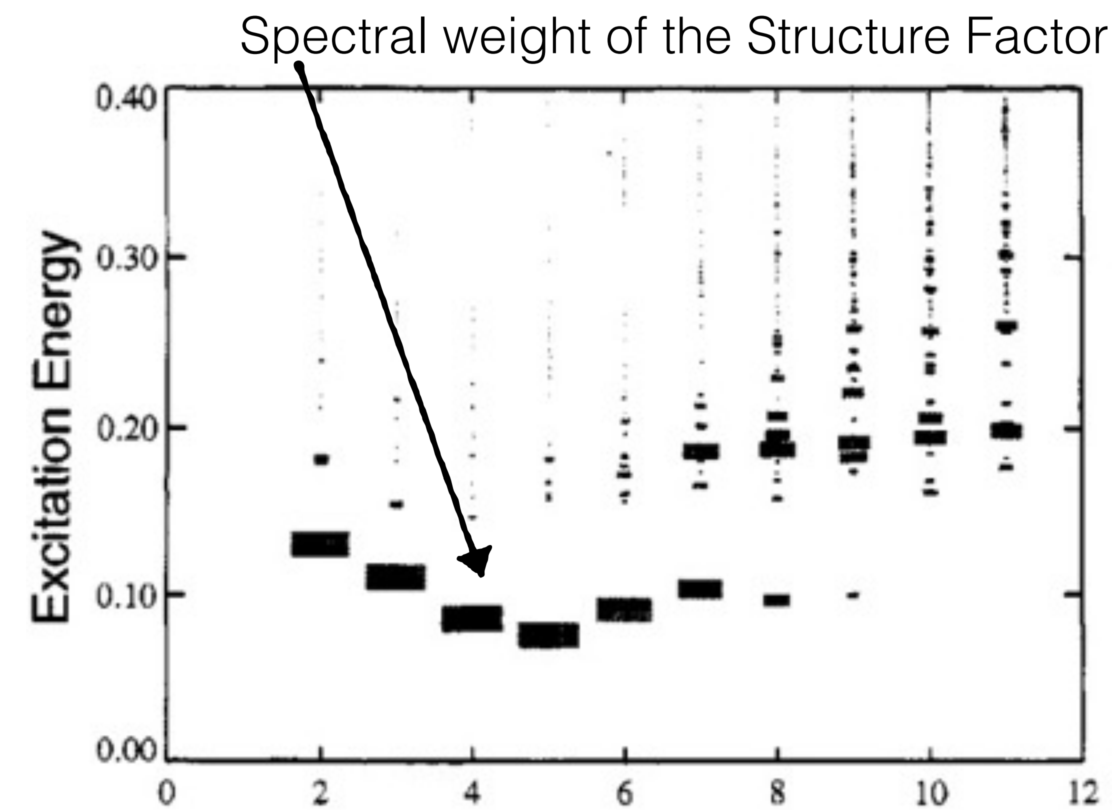
SINGLE MODE APPROXIMATION (SMA)

SMA states that observables are saturated by $\bar{\rho}(\mathbf{k})|0\rangle$

For example, optical absorption spectrum $\Delta(k) = \dfrac{\langle 0|\bar{\rho}(-\mathbf{k})H\bar{\rho}(\mathbf{k})|0\rangle}{\langle 0|\bar{\rho}(-\mathbf{k})\bar{\rho}(\mathbf{k})|0\rangle}$



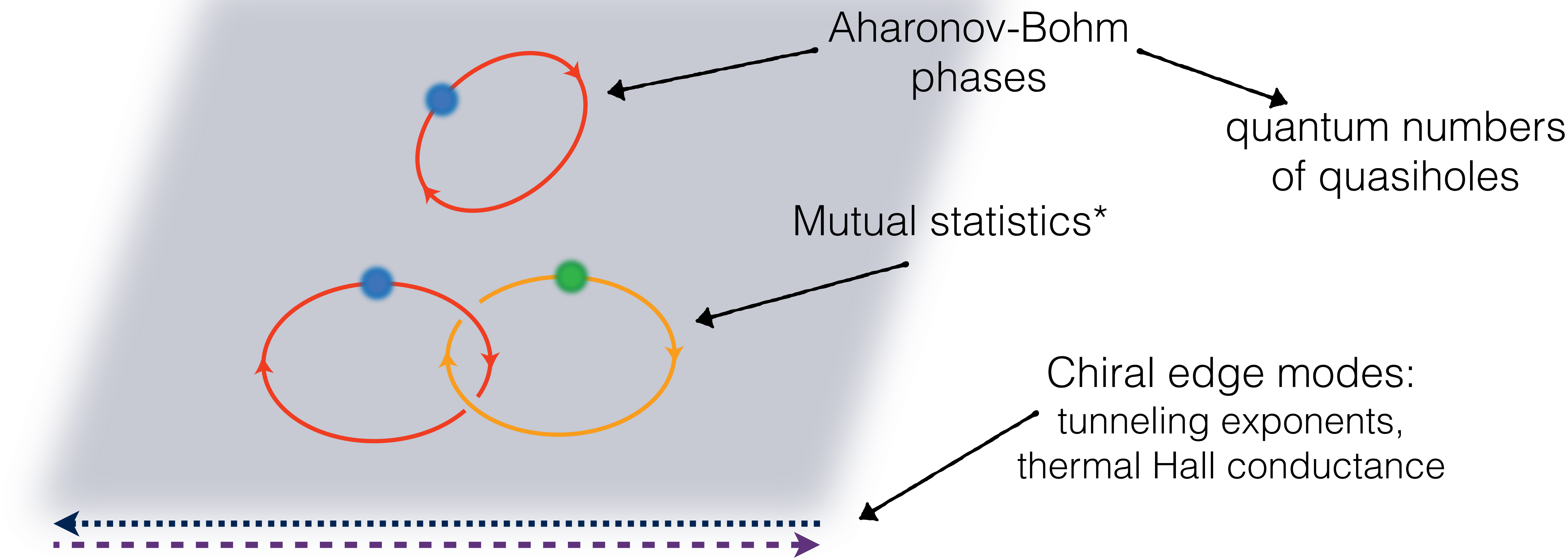
Repellin, Neupert, Papic, Regnault 2014



Platzman, He 1996

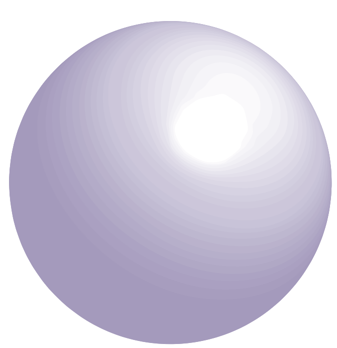
SMA is *accurate* at small $\mathbf{k}$

SMA is **exact** near the nematic phase transition



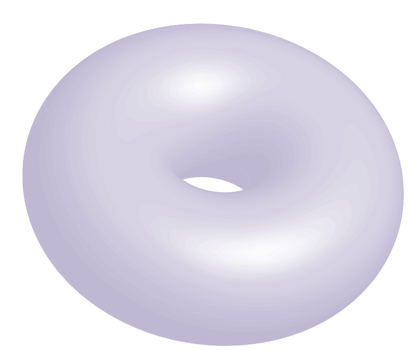
$$W[A,\omega] = \int \mathcal{D}a e^{iS[a;A,\omega]}$$

Linear response:
Hall conductance, Hall viscosity,…



Shift $\mathcal{S} = 2s$

$$N = \nu N_\phi + \nu \mathcal{S}$$

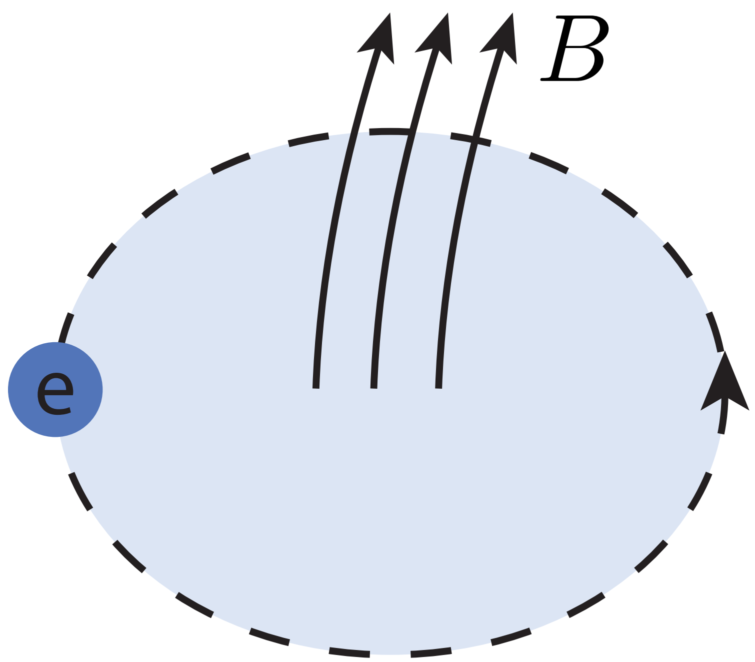


Ground state degeneracy k

*after AB phases are subtracted

AHARONOV - BOHM EFFECT

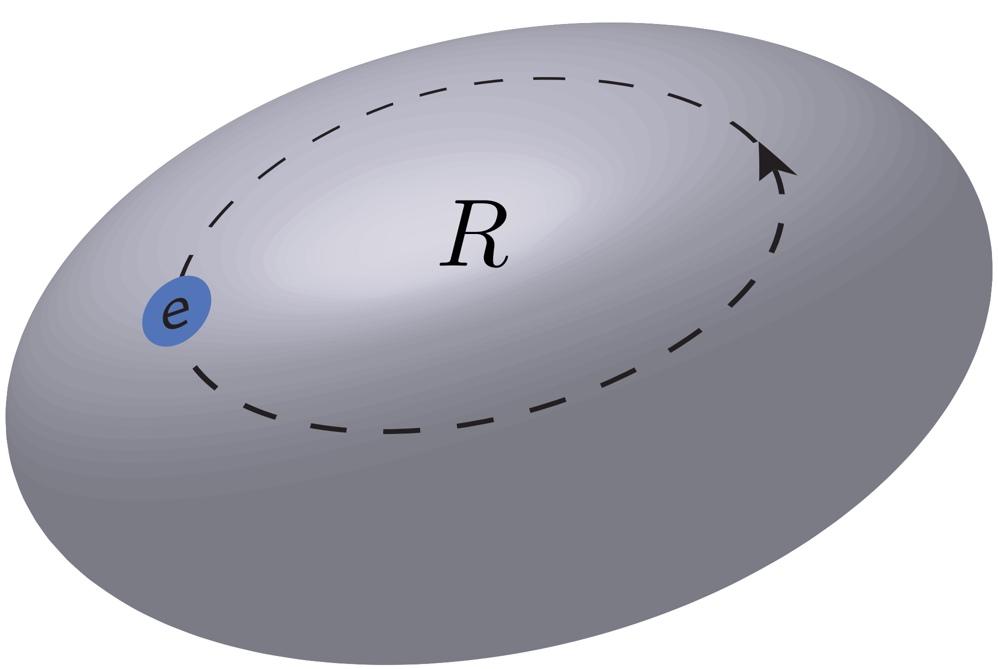
Electrically charged particles in magnetic field have AB effect



$$\Psi \to \exp\left(2\pi i e \oint_{\mathcal{C}} A_i dx^i\right)\Psi$$

$$\partial_1 A_2 - \partial_2 A_1 = B$$

Particles with orbital spin in curved space have AB effect



$$\Psi \to \exp\left(2\pi i \bar{s} \oint_{\mathcal{C}} \omega_i dx^i\right)\Psi$$

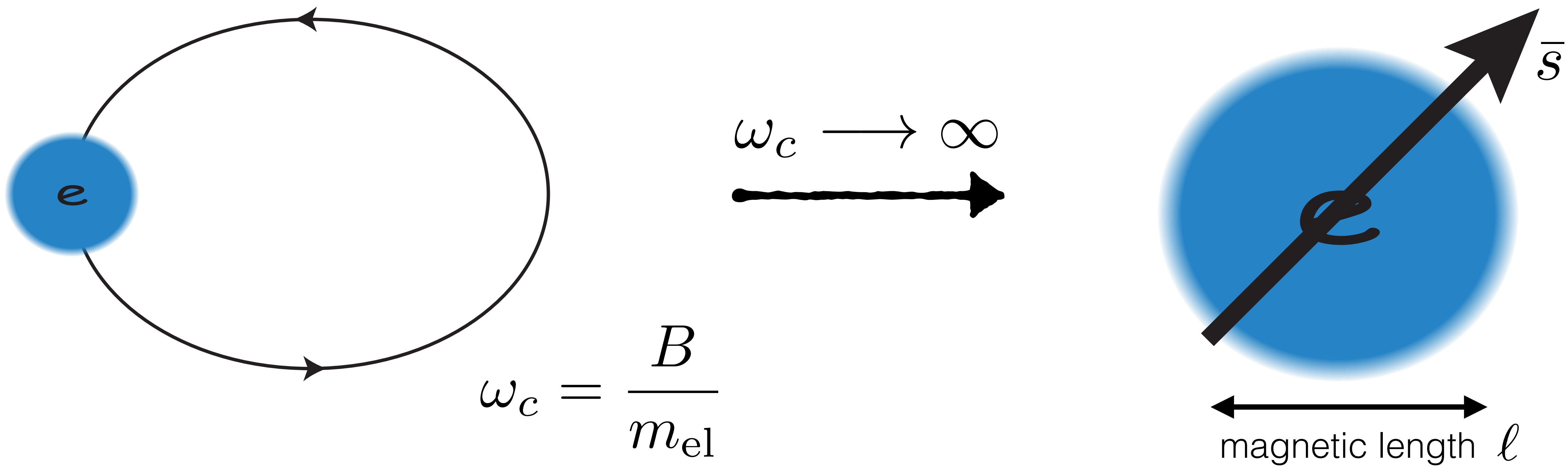
$$\partial_1 \omega_2 - \partial_2 \omega_1 = R/2$$

Wen Zee 1992

ORBITAL SPIN

In the remainder of the talk I will use the term ``orbital spin''.

In magnetic field electrons quickly move in cyclotron orbits



We consider the limit $m_{\rm el} \longrightarrow 0$

``Orbital spin'' describe the coupling of the low energy physics to spatial geometry

Read 2009

Wen Zee 1992

COMPOSITE FERMI LIQUID

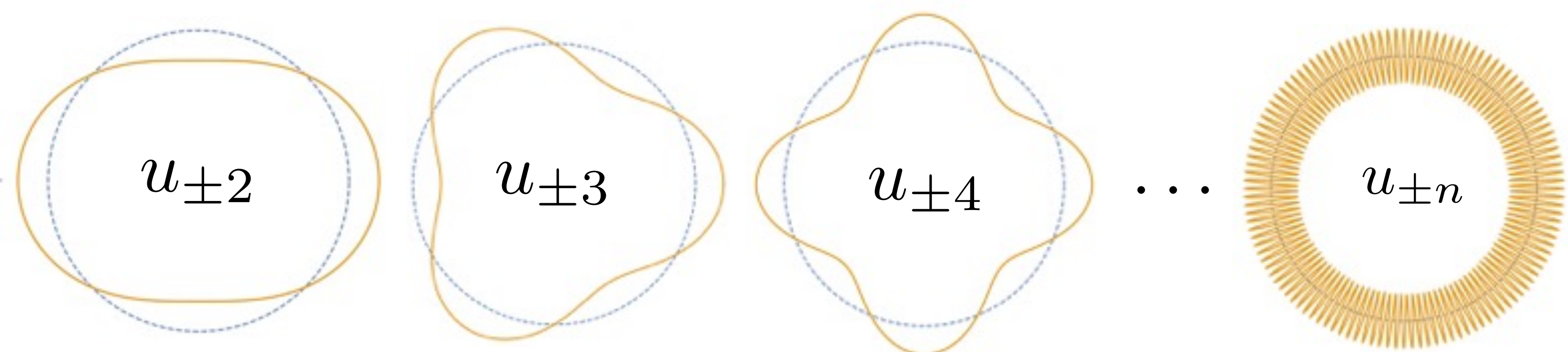
States at filling $\nu = \frac{N}{2N+1}$ $\approx$ IQH of composite fermions at $\nu_{\text{eff}} = N$

Can be treated via Fermi liquid theory when N is large

Semiclassically the d.o.f. are multipolar distortions of the Fermi surface

Dilation   Translation

non-dynamical



$\cdots$ dynamical

Shear

``Higher spin" area preserving deformations

Golkar Nguyen Roberts Son 2016

COMPOSITE FERMI LIQUID IN SMA

Hamiltonian

$$H = \frac{v_F k_F}{4\pi} \sum_n \int d^2\mathbf{x}(1 + F_n) u_n(\mathbf{x}) u_{-n}(\mathbf{x}),$$

Phenomenological ``Landau parameters''

CCR

$$[u_n(\mathbf{x}), u_m(\mathbf{x}')] = \frac{2\pi}{k_F^2}\left(n\bar{b}\delta_{n+m,0} - ik_F\delta_{n+m,1}\partial_{\bar{z}} - ik_F\delta_{n+m,-1}\partial_z\right)\delta(\mathbf{x} - \mathbf{x}')$$

All modes are gapped at $\Delta_n = n(1 + F_n)\omega_c$

The limit $\Delta_2 \ll \Delta_n$ for all $n \geq 3$ is the SMA

Only dynamics of shear distortions $u_{\pm 2}$ remains

$$[u_2(\mathbf{x}), u_{-2}(\mathbf{x}')] = \frac{4\pi}{2N+1}\delta(\mathbf{x} - \mathbf{x}')$$

Same as linearized bimetric

Nguyen, AG, Son In Progress

COMPOSITE FERMI LIQUID IN SMA II

Effective Lagrangian in SMA

$$\mathcal{L}_{\rm SMA} = -\frac{i}{2}\frac{2N+1}{2\pi}u_2\dot{u}_{-2} + \frac{i}{2}\frac{N^2(2N+3)\ell^2}{12\pi}u_2\Delta\dot{u}_{-2} - \frac{c_0(2N+1)\omega_c}{2\pi}u_2u_{-2} + \frac{c_2(2N+1)\omega_c\ell^2}{2\pi}u_2\Delta u_{-2}$$

coincides with the linearized bimetric theory

$$\mathcal{L}_{\rm bm} = \frac{2N+1}{16\pi}Ad\hat{\omega} - \frac{N^2(2N+3)}{96\pi}\hat{\omega}d\hat{\omega} - \frac{\tilde{m}}{2}\left[\hat{g}_{ij}g^{ij} - \gamma\right]^2 - \frac{\alpha}{4}\left[\Gamma - \hat{\Gamma}\right]^2$$

Bimetric theory prescribes coupling of the CFL to curved space

<u>Conjecture:</u>

Bimetric theory is the geometric non-linear completion of the CFL in the SMA.

What about beyond SMA ?